TECHNICAL NOTE

Predicting the performance of a heat-pipe heat exchanger, using the effectiveness–NTU method

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An analysis of the thermal performance of a heat-pipe heat exchanger based on the effectiveness–NTU model is developed. The predicted thermal performance using this model is compared with results obtained by previous investigators. Good agreement was observed.

Keywords: heat exchangers; heat pipe; (NTU) method

Introduction

The special features of heat pipes have made them attractive for use as heat-pipe heat exchangers, especially as waste energy recovery devices.[1] Attempts to predict the performance of heat-pipe heat exchangers using a conductance model were made by Amode and Feldman,[2] and Lee and Bedrossian.[3] The effectiveness–NTU method has also been used to predict the performance of a heat-pipe heat exchanger. Expressions of effectiveness for a single heat pipe, and for n rows of a heat-pipe heat exchanger were reported by Krishman and Rao,[4] and Chaudourne.[5] However, most of the work published earlier concentrated on theoretical studies and presented no experimental data. Huang and Tsuei[6] used the conductance model to analyze heat-pipe heat exchanger performance and compared the predicted results to their experimental results, for which they obtained good agreement. However, their method of analysis requires iterative procedures to make the values of parameters that were assumed initially converge to a specified accuracy. This difficulty can be overcome by the effectiveness–NTU method presented in this paper. The outlet temperature, as well as the temperatures between the rows of hot and cold fluids, could be readily calculated without the iterative procedure. The results calculated using the model were almost identical with those presented by Huang and Tsuei.[6] They showed that the effectiveness–NTU method is a viable alternative method for predicting the performance of a heat-pipe heat exchanger but requiring much less computation.

Modeling the heat-pipe heat exchanger

A heat-pipe heat exchanger can be considered to be a liquid-coupled indirect regenerator, with the heat pipes providing the coupling. Overall effectiveness can be expressed as a function of two separate heat exchangers: one with evaporation and the other with condensation of the coupling fluid. The corresponding effectiveness of the evaporator section is

$$\varepsilon_e = 1 - e^{-\mathrm{NTU}_e} \tag{1}$$

and that of the condenser section is

$$\varepsilon_c = 1 - e^{-\mathrm{NTU}_c} \tag{2}$$

where NTU_e and NTU_c are the number of transfer units of the evaporator section and condenser section, respectively. The number of transfer units of the two sections can be written as

$$\mathrm{NTU}_e = \frac{U_e A_e}{C_e} \tag{3}$$

and

$$\mathrm{NTU}_c = \frac{U_c A_c}{C_c} \tag{4}$$

where U_e and A_e are the thermal conductance and area of the evaporator section, respectively, and C_e is the thermal capacity of the fluid passing through the evaporator section. The corresponding values of those quantities in the condenser section are denoted by U_c, A_c, and C_c.

The overall effectiveness of a single-heat-pipe heat exchanger can be generalized as,[7]

$$\varepsilon_p = \left(\frac{1}{\varepsilon_{\min}} + \frac{C}{\varepsilon_{\max}}\right)^{-1} \tag{5}$$

where C is the capacity ratio and $\varepsilon_{\min}$ and $\varepsilon_{\max}$ are the effectiveness of minimum and maximum fluid, respectively. Figure 1 shows the plot of $\varepsilon_p/\varepsilon_{\min}$ versus $\varepsilon_{\min}/\varepsilon_{\max}$ for various values of C. When $\varepsilon_{\min}/\varepsilon_{\max}$ is very small, $\varepsilon_p/\varepsilon_{\min}$ approaches unity.

A unit of heat-pipe heat exchanger may consist of n rows and heat pipes, which can be considered as n heat exchangers connected in series.[8] The effectiveness of such a heat exchanger in counter flow can be written as

$$\varepsilon = \frac{1 - \left(\dfrac{1 - C\varepsilon_p}{1 - \varepsilon_p}\right)^n}{C - \left(\dfrac{1 - C\varepsilon_p}{1 - \varepsilon_p}\right)^n} \tag{6}$$

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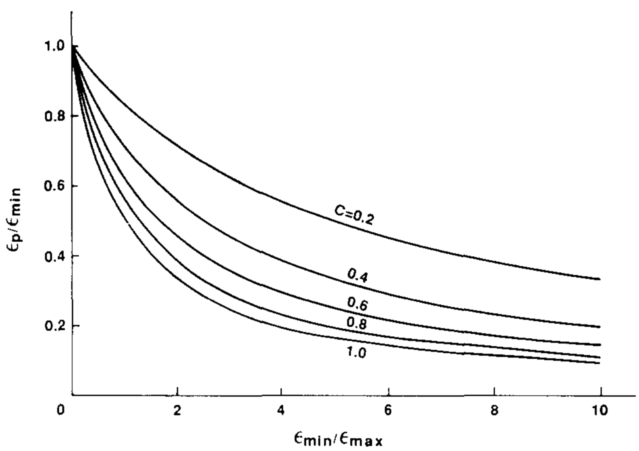

Figure 1 Effectiveness of a single heat pipe

Knowing the number of rows, n, effectiveness can be plotted in terms of ε_p and C. The plot of $n=4$ is shown in Figure 2.

If the thermal capacities of the two fluids are known, the effectiveness of the system can be written in terms of temperature ratio. For example, if hot fluid is the minimum fluid, ε takes the form:

$$\varepsilon=\frac{T_{hi}-T_{ho}}{T_{hi}-T_{ci}} \tag{7}$$

where T_{hi} and T_{ho} are the inlet and outlet temperatures of the hot fluid, respectively, and T_{ci} is the inlet temperature of the cold fluid. If the two inlet temperatures are specified, Equations 6 and 7 can be used to calculate the outlet temperatures of hot and cold fluids. As ε_p is a function of ε_{min}, ε_{max}, and C, the conductance of evaporator and condenser sections must be known. The steps for computing the thermal conductance was detailed in DSDU 79012.[9] However, the results were intended to be used for comparison with Huang and Tsuei's experimental data,[6] so a simplified model of theirs was adopted for the present calculation. In their model, they identified three components of total conductance: (1) thermal conductance from the hot fluid to the wall of the evaporator of the pipe, $(hA)_h$; (2) thermal conductance from the surface of the evaporator to the surface of the condenser, through the wall, wick, and vapor flow of the pipe, $(UA)_p$; and (3) the thermal conductance from the surface of the condenser to the cold fluid is denoted by $(hA)_c$. Huang and Tsuei used a heat exchanger consisting of four rows of heat pipes in a staggered arrangement. There were 8 pipes per row; each pipe was 33.7 mm in diameter and had an effective length of 610 mm. The longitudinal and transverse pitches were 44.7 mm and 38.7 mm, respectively. The partition plate was fixed in the middle of the pipes to give equal areas of evaporator and condenser. The $(UA)_p$ of the pipe used was found experimentally to be 3.36 W/°C. A reasonable assumption is that the thermal conductance from the outer surface of the evaporator to the vapor region of the heat pipe, $(UA)_{pe}$ is equal to that from the vapor region of the heat pipe to the outer surface of the condenser, $(UA)_{pc}$. The thermal conductance in various parts of the heat pipe is shown in Figure 3.

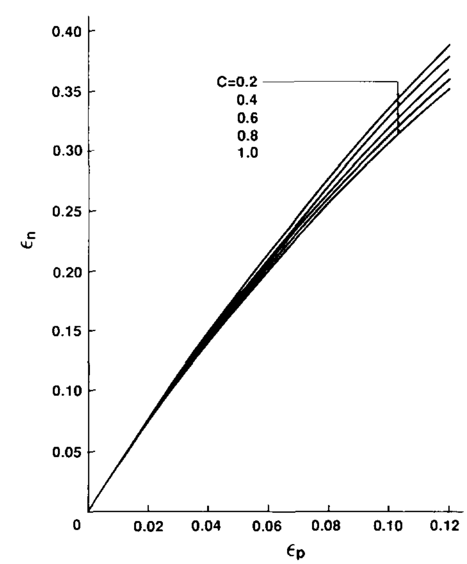

Figure 2 Effectiveness of a heat-pipe heat exchanger with four rows

Notation

A	Area
C	Capacity ratio
C_c	Thermal capacity of cold fluid
C_e	Thermal capacity of hot fluid
C_v	Void fraction of heat-pipe heat exchanger
D_p	Characteristic diameter of pipe
k	Conductivity of fluid
n	Number of rows
NTU	Number of transfer units
Pr	Prandtl number
Re	Reynolds number
T	Temperature
U	Overall heat transfer coefficient
u	Characteristic flow velocity

Greek symbols

ε	Effectiveness of heat-pipe heat exchanger
ε_c	Effectiveness of condenser section of a heat pipe
ε_e	Effectiveness of evaporator section of a heat pipe
ε_p	Effectiveness of a single heat pipe
ρ	Fluid density
μ	Fluid viscosity

Subscripts

c	Condenser section of heat pipe
ci	Condition of the cold fluid inlet
co	Condition of the cold fluid outlet
$c1$	Condition of the cold fluid downstream from first row
e	Evaporator section of heat pipe
hi	Condition of the hot fluid inlet
ho	Condition of the hot fluid outlet
$h1$	Condition of the hot fluid downstream from first row
pe	Condition only for heat-pipe evaporator section
pc	Condition only for heat-pipe condenser section
max	Condition of the maximum fluid
min	Condition of the minimum fluid

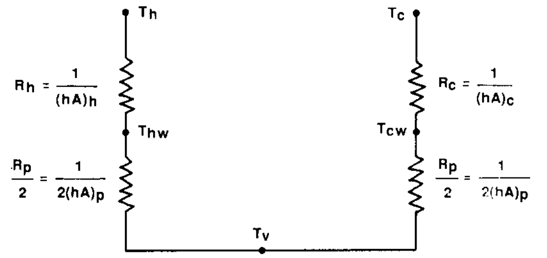

Figure 3 Thermal resistance network across a single heat pipe

Thus the total thermal conductance in the evaporator region is

$$(UA)_e=\left(\frac{1}{(UA)_{pe}}+\frac{1}{(hA)_h}\right)^{-1} \tag{8}$$

and that in the condenser region is

$$(UA)_c=\left(\frac{1}{(UA)_{pc}}+\frac{1}{(hA)_c}\right)^{-1} \tag{9}$$

The convective heat transfer coefficients, h_h and h_c, can be obtained by the universal correlation presented by Whitaker:[10]

$$\mathrm{Nu}=f(0.5\mathrm{Re}^{1/2}+0.2\mathrm{Re}^{2/3})\mathrm{Pr}^{1/3}(\mu/\mu_w)$$

where Nu and Re are defined as

$$\mathrm{Nu}=\left(\frac{hD_p}{k}\right)\frac{C_v}{1-C_v} \quad \text{and} \quad \mathrm{Re}=\left(\frac{\rho D_p u}{\mu}\right)\frac{C_v}{1-C_v}$$

The characteristic diameter, D_p, is 1.5 times the outside diameter of the pipe for the tube bank, C_v is the void fraction of the heat-pipe heat exchanger, and u is the characteristic flow velocity, taken as the maximum velocity in the tube bank; ρ, k, and μ are the density, conductivity, and viscosity, respectively, of the fluid; f is a parameter that depends on the number of rows in the heat-pipe heat exchanger and the geometrical arrangement of the pipes; and Nu, Re, and Pr are the Nusselt, Reynolds, and Prandtl numbers, respectively.

In Equations 8 and 9, both $(UA)_{pe}$ and $(UA)_{pc}$ equal 6.72. This magnitude is valid only for the heat pipe used by Huang and Tsuei[6] and is subject to the condition that the evaporator and condenser sections are of equal length.

Results and discussion

The merit of using the effectiveness–NTU method to calculate a heat-pipe heat exchanger is that it does not require iteration. Thus much less computing time is required. Knowing the mass flow rates of hot and cold fluids and the geometric parameters of heat-pipe heat exchangers allows calculation of the convective heat transfer coefficients by Whitaker's correlation equation.[10] As Equations 5 and 6 were derived for a single heat pipe with n rows, the mass flow rate was normalized by dividing the total mass flow rate by the number of heat pipes in the row. The effectiveness of a single heat pipe and the total effectiveness of a heat-pipe heat exchanger in counter flow can be calculated by Equations 5 and 6, respectively. The outlet temperature of the minimum fluid can be calculated from Equation 7. The energy-balance equation is used to compute the outlet temperature of the maximum fluid:

$$(MC_p)_h(T_{hi}-T_{ho})=(MC)_c(T_{co}-T_{ci}) \tag{10}$$

The results calculated were very close to Huang and Tsuei's theoretical results, obtained by using the conductance model with iteration.[6] The heat transfer rate can be calculated from known inlet and outlet fluid temperatures. Figure 4 shows the comparison of our results with their experimental values. Note that the agreement is reasonable.

Having obtained outlet fluid temperatures, we can use Equation 5 and the heat balance equation to calculate the temperature between different rows of the heat exchanger. The calculation can start with the first row and then proceed to the downstream rows.

The effectiveness of the heat pipe was calculated from Equation 5. If hot fluid is the minimum fluid, ε_p is

$$\varepsilon_p=\frac{T_{hi}-T_{h1}}{T_{hi}-T_{c1}}$$

With the aid of the energy-balance equation of the first row of the heat-pipe heat exchanger,

$$(MC_p)_h(T_{hi}-T_{h1})=(MC_p)_c(T_{co}-T_{c1})$$

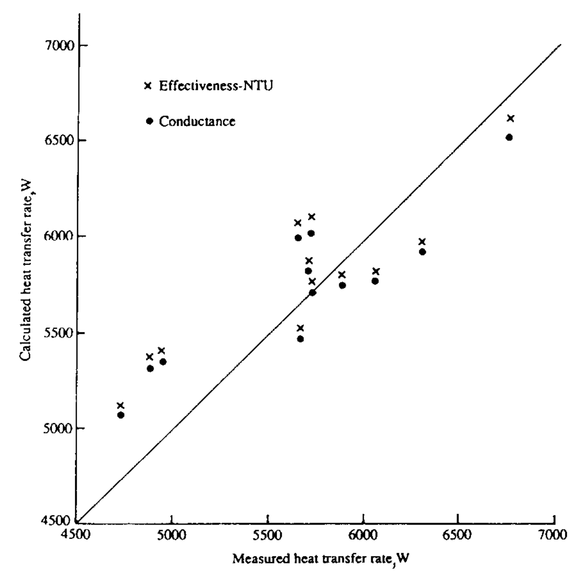

Figure 4 Comparison of the heat transfer rate

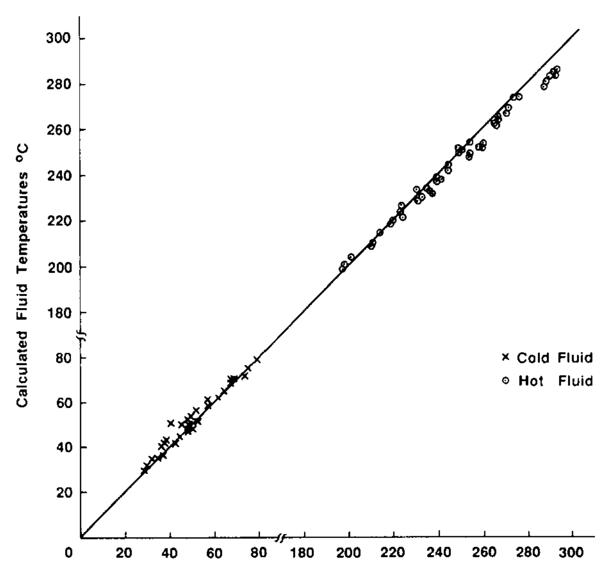

Figure 5 Comparison of fluid temperatures between rows

the temperatures of the hot fluid, T_{h1}, and cold fluid, T_{c1}, downstream from the first row can be calculated. Thus the temperature distribution of the hot and cold fluids across each row of the heat-pipe heat exchanger can be determined. Our results all were very close to those obtained by Huang and Tsuei.[6] The reason could be that the same methods were used to calculate the thermal conductance. The flow rates of hot fluid in our calculation were based on the specified temperature instead of at standard conditions, as were their theoretical results. Our results were also compared against the results of the 12 experiments reported by Huang and Tsuei.[6] In those experiments, the flow rates and temperatures of air used as the cold and hot fluids were varied to obtain a range of heat transfer between 6,750 W to 4,720 W. Figure 5 shows good agreement. However, our calculation procedure is not iterative, which greatly reduces computing time.

Conclusion

The good agreement between our results and those calculated using a conductance model indicated that the effectiveness–NTU is a convenient method of predicting heat-pipe heat exchanger performance. The temperature distribution of both hot and cold fluids can be calculated readily without iteration.

References

1 Chi, S. W. *Heat Pipe Theory and Practice*, McGraw-Hill, New York, 1976

2 Amode, J. O. and Feldman, K. T. Preliminary analysis of heat pipe heat exchangers for heat recovery. ASME Paper No. 75-WA/HT-36, 1976

3 Lee, Y. and Bedrossian, A. The characteristics of heat exchanger using heat pipes or thermosyphons. *Int. J. Heat Mass Transfer*, 1976, **21**, 221–229

4 Krishman, K. N. and Rao, K. S. Analysis and design of a heat pipe heat exchanger. *Proc. 5th Int. Heat Pipe Conf.*, Tsukuba Science City, Japan, 1984, 152–156

5 Chaudrone, S. Modelization and optimization of heat pipe heat exchangers. *Proc. 5th Int. Heat Pipe Conf.*, Tsukuba Science City, Japan, 1984, 53–58

6 Huang, B. J. and Tsuei, J. T. A method of analysis for heat pipe heat exchangers. *Int. J. Heat Mass Transfer*, 1985, **28**, 553–562

7 Kays, W. M. and London, A. L. *Compact Heat Exchangers*, 3rd ed., McGraw-Hill, New York, 1984

8 Domingos, J. D. Analysis of complex assemblies of heat exchangers. *Int. J. Heat Mass Transfer*, 1969, **12**, 537–548

9 DSDU 79012. Heat pipes performance of capillary-driven designs. Engineering Sciences Data Unit, London, 1979

10 Whitaker, S. Forced convection heat transfer correlations flow in pipes, past flat plates, single cylinders, single spheres, and for flow packed beds and tube bundles. *J. AIChE*, 1972, **18**(2), 361–371